THEORETICAL ASPECTS OF TRANSIENT ELECTROMAGNETIC FIELD IN FINITE SIZED CONDUCTING MEDIA

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INTRODUCTION

It is generally accepted that electromagnetic disturbances diffuse into the bulk region of highly conducting media instead of propagating with wave-like characteristics [1]. This can be explained based on the fact that the high frequency components of the electromagnetic field decay rapidly, leaving the electromagnetic state in the bulk material quasistatic. For the application of this phenomena to practical testing, Ross et al. developed a formalism describing the diffusion of electromagnetic field in a finite thickness conductor and demonstrated the effect of thickness on the time rate of damping of field amplitude [2].

Certain experimental results reported in the past, however, can be explained based on the wave-like propagation of electromagnetic disturbances through the bulk region of a conductor. Cavcey has investigated the electromagnetic pulse transmission through aluminum plates and obtained the thickness dependence of time delay in peak position which seems to be consistent with wave-like propagation [3]. The detected pulses also clearly showed a broadening, which is perhaps due to dispersion, strengthening the interpretation of the wave-like nature of pulse propagation. Gibbs and Campbell obtained the depth information of flaws in different locations along the thickness of aluminum plates based on the propagation and reflection of electromagnetic pulses in a conducting medium where a discontinuity in characteristic impedance occurs [4].

The basic inversion mechanism depends on the nature of the electromagnetic field propagating in the test media and thus it is critical to clarify the issues. In this paper, we first review the basic elements of both wave propagation and diffusion, and discuss the strengths and weaknesses of each point of view in comparison with the experimental results available in the literature.

BRIEF REVIEW OF MATHEMATICS OF WAVE AND DIFFUSION-LIKE PROPAGATION OF DISTURBANCE

The one dimensional linear wave equation has the following simple form:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c_o^2}\frac{\partial^2 y}{\partial t^2}$$

where c_o is the phase velocity of the traveling wave as will be shown clearly in the following. A simple and illustrative way of solving the above equation was due to D'Alembert (1747) and will be briefly described in this section [5].

Introducing the change of variables as $\xi = x - c_o t$ and $\eta = x + c_o t$, one can readily obtain the following expression:

$$\frac{\partial^2 y}{\partial\xi\partial\eta} = \frac{\partial}{\partial\xi}\left(\frac{\partial y}{\partial\eta}\right) = 0.$$

From this expression, it is clear that $\frac{\partial y}{\partial\eta}$ is independent of ξ, so that

$$\frac{\partial y}{\partial\eta} = \phi(\eta)$$

where $\phi(\eta)$ is an arbitrary function of η. Integrating over ξ yields

$$y(\xi,\eta) = \int^{\eta}\phi(\eta')d\eta' + f(\xi)$$

where $f(\xi)$ is an arbitrary function of ξ. The integration yields another arbitrary function, say, $g(\eta)$, and the final expression of $y(\xi,\eta)$ is obtained to be

$$y(\xi,\eta) = f(\xi) + g(\eta).$$

Written as a function of original variables, the solution of wave equation is as follows:

$$y(x,t) = f(x - c_o t) + g(x + c_o t).$$

According to the above equation any arbitrary form of disturbance will travel both in the $\pm$ x directions at a speed of c_o without diminishing its amplitude or distorting its shape in a nondispersive and nondisspative medium.

The same results can be obtained by applying a Green's function method [6]. The Green's function for displacement due to an external δ - function impact is

$$g(x/\varsigma;t) = \frac{1}{2}\delta(\varsigma - x + ct) + \frac{1}{2}\delta(\varsigma - x - ct).$$

The above solution reduces to $\delta(x\text{-}\zeta)$ at $t = 0$. To include the effect of realistic input impulse of finite width and height, one needs to integrate as

$$y(x,t) = \int_{-\infty}^{t} g(x/\varsigma;t,\tau)f(\tau)d\tau$$

where *f(t)* is the actual functional form of the impulse. Fig. 1 is a schematic representation of wave propagation where two wave packets are generated by an external impulse and travel in the opposite directions in a nondispersive, nondissipative medium.

We, now, turn our attention to diffusion phenomena. The one-dimensional diffusion equation has the following form:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{a^2}\frac{\partial y}{\partial t}$$

The time dependence of the disturbance of any field produced by an externally applied impulse can be most easily solved by a Green's function method applied to a one-dimensional thermal diffusion problem as treated in detail in Ref. 6. Through simple mathematical steps, one can easily obtain a solution of the heat conduction equation with arbitrary initial condition. This is exactly equivalent to the case of electromagnetic field diffusion problem in one-dimensional semi-infinite medium where the solution is

$$y(x,t) = \frac{1}{\sqrt{4\pi a^2 t}} \int_{-\infty}^{+\infty} e^{-(x-\xi)^2/4a^2t} f(\xi)d\xi .$$

Fig. 2 shows the time dependence of *y(x,t)*. The characteristic distribution of a diffusing field is that the position of peak amplitude is stationary, whereas the peaks of the traveling waves move along the packets as seen in Fig. 1.

DIFFUSED FIELDS; THROUGH TRANSMISSION

For the purpose of discussion, we consider the region x > 0 as an electrically conducting bulk material occupying the semi-infinite space. Fig. 3 shows a detailed view of the field distributions in the region of interest. Suppose the field amplitude is measured at location A. It takes a finite time for the field amplitude to reach a measurable level and, as time elapses, the field amplitude at this location increases to a maximum and will then begin to decrease. If we repeat this measurement at a point B which is farther from the impulse location than A, the first detection of measurable field amplitude at this location occurs at a later time than it was observed at A. Likewise, all the other events in the time scale observed at location B related to the field amplitude will be delayed with respect to those of A. Consequently, the location of the peak in the field amplitude observed at B will be delayed with respect the that observed at A. One can also infer, by a visual inspection, that the detected waveform broadens as the observation point moves farther.

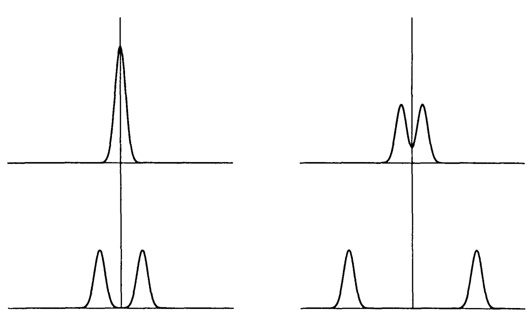

Fig. 1. Schematic representation of two wave packets traveling in opposite directions produced by an impulse of finite width and height applied at $t = 0$ and $\zeta = 0$. The gaussian waveforms are calculated for $t = 0$, 2, 4 and 12 in the unit of $1/c_o$.

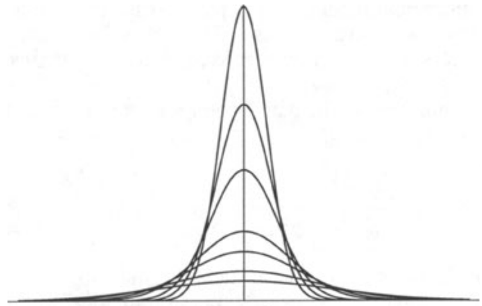

Fig. 2. Time dependence of the spatial distribution of disturbance produced by an impulse applied at the central location. The curves are calculated for t = 0.02, 0.03, 0.045, 0.085, 0.12, 0.2 and 0.3 sec with a = 0.1 m/sec$^{1/2}$.

Fig. 4 shows the field amplitudes as a function of time observed at several different locations. It is seen that the peaks become subtle as the observation point moves away from the origin. This is due to the slow damping of the field amplitude in time which was caused by not counting the eddy current loss. For an accurate evaluation, one has to take into account that the rate of eddy current loss is proportional to $(\partial y / \partial t)^2$. From this one can immediately see that the rate of eddy current loss has an explicit spatial dependence. The inclusion of this effect in the field amplitude observed at different locations as a function of time turned out be to overly complicated. Instead, it will be demonstrated in the following that the appearance of the peaks in the field distribution over time can be accomplished by taking the time dependence of the eddy current loss only.

To reflect the time rate of loss, an exponential damping term, i. e., $e^{-\gamma t}$, with γ being an adjustable parameter, is multiplied to the original solution of the diffusion equation, which is a gaussian distribution. The results are shown in Fig. 5 where the peak appears in all the distributions of the field amplitudes.

ISSUES RELATED TO INVERSION PROBLEMS IN PULSED EDDY CURRENT TESTS

Solving the diffusion equation to obtain the results of Fig. 4, the conducting medium has been considered as a semi-infinite block and, consequently, the only boundary condition used was $y(\infty) = 0$. For a qualitative comparison purpose, however, one can assume that the solution obtained for a semi-infinite conductor can be applied to the description of fields in finite conductor since the diffusing field is not as sensitive to the presence of boundary as the field of traveling wave is. Hence, the locations for field observation in the conductor are taken as the opposite boundaries of conductors of finite thicknesses, and the results of Fig. 5 are taken as the waveform of the magnetic field detected at these boundaries. Those results are very similar to the experimental results of Cavcey [2] which show the delay in peak location that is consistent with the increased pathlength of the magnetic pulse [7] as the thickness of the aluminum block increases.

The simple example given above demonstrates that, when detected at the opposite side of bulk conducting plate, purely diffusing magnetic fields can provide results which are very similar to those expected from transmission of traveling waves through the plate. For almost all the practical situations that involve pulsed eddy current tests, however, one has to detect the signals at the same side, i. e., borrowing terms from ultrasonics, the test configuration must be pulse-echo mode instead of pitch-catch mode. To evaluate the thickness of a conducting plate in pulse-echo mode, one may immediately relate the

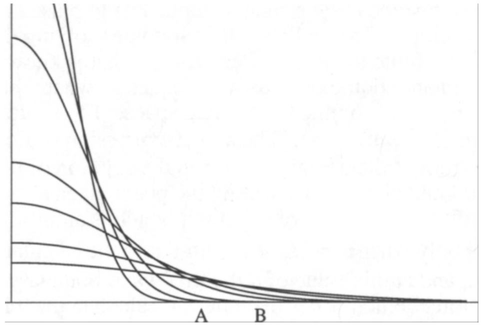

Fig. 3. Zoomed in view of spatial distributions of the diffusing field amplitude at different times in the region of interest, i. e., $x > 0$. It takes a longer time for the field amplitude to reach a certain level at B than at A.

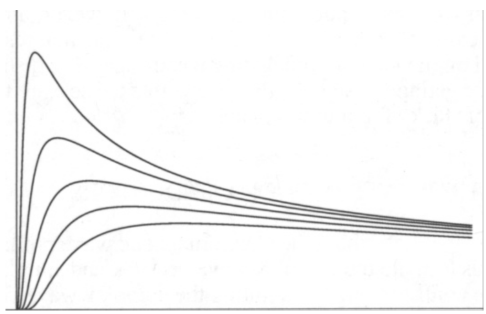

Fig. 4. Field amplitudes as a function of time observed at different locations without including the eddy current loss. The five locations are $x = 0.318$, 0.477, 0.637, 0.796 and 0.955 with $a = 1$.

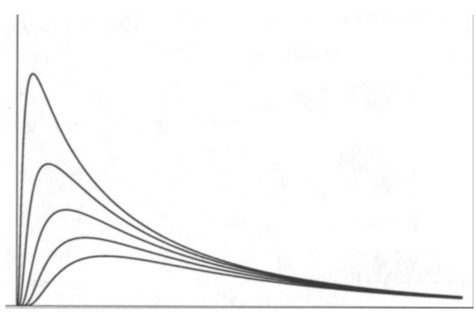

Fig. 5. Field amplitudes as a function of time observed at different locations including the eddy current loss which was approximated as $e^{-\delta t}$ with $\gamma = 0.5$.

detection of signals reflected from the opposite boundary. The problem is, however, that one cannot define a characteristic impedance of a diffusing field such that there is no definable impedance mismatch at the boundary.

In an effort to provide the basic principle applicable to pulsed eddy current test method, Ross et al. developed a formalism which involves solving the diffusion equation in a conducting plate of a finite thickness. Performing what is known as the inverse-q transform, they obtained an inhomogeneous wave equation where the parameter q in the transformed space corresponds to time t in the real space. The solution of the wave equation is in the form of $\sin(\omega q - kx)$. The actual expression of frequency in this wave equation is the square-root of the damping factor in the solution of the diffusion equation which is the most important element connecting the phenomena of both spaces. One can easily show that the solution of the wave equation, which is an infinite series, has meaningful amplitude only when $q = 2L\sqrt{\mu\sigma}$, similar to the calculation of the structure factor of a crystal [8], and at this value of q the first wave-boundary interaction is felt. This formalism, therefore, demonstrates that one can calculate the thickness of plate L from the damping rate of the diffusing field measured at $x \leq 0$ without requiring the physical reflection at the boundary at x = L.

The combination of the results of Fig. 4 and 5, and the formalism of Ross et al. provides a strong evidence supporting the diffusion of magnetic field as the base of pulsed eddy current method. The issue involving wave-like propagation or diffusion of electromagnetic fields through a conducting medium is, however, unresolved since the elements of the above discussion cannot explain the experimental results of Gibbs and Campbell [3]. Based on the idea of propagating electromagnetic disturbance with a finite speed and by properly gating the signals, they were able to measure the location of the flaw along the thickness of stack of aluminum plates.

ELECTROMAGNETIC WAVES IN GOOD CONDUCTORS

A traditional way of describing the electromagnetic wave propagation in conducting or dissipative media is to write the complex wave vector k and simplify it by omitting the terms with negligible contribution. Substituting the complex wave vector back into the original wave equation, one obtains

$$\vec{E} = \vec{E}_o e^{-\alpha x} e^{i(\omega t - \beta x)}$$

where β and α are the real and imaginary components of complex wave vector k, respectively. The above equation describes an electric wave whose amplitude is rapidly damping as it propagates into a conducting medium. Also, for a good conductor the wave equation reduces to a diffusion equation

$$\nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0 \text{ providing } k^2 = -i\omega\sigma\mu$$

and the following expressions are readily obtained:

$$k = \beta - i\alpha = \left(\frac{\omega\sigma\mu}{2}\right)^{\frac{1}{2}}(1-i) = \frac{1-i}{\delta}, \ \beta = \frac{2\pi}{\lambda} = \alpha = \frac{1}{\delta} = \left(\frac{\omega\sigma\mu}{2}\right)^{\frac{1}{2}}$$

where δ is the skin depth. Also, it is shown that, in a good conductor, the characteristic impedance is

$$Z = \frac{E}{H} = \frac{\omega\mu}{k} = \left(\frac{\omega\mu}{\sigma}\right)^{\frac{1}{2}} e^{\frac{i\pi}{4}}$$

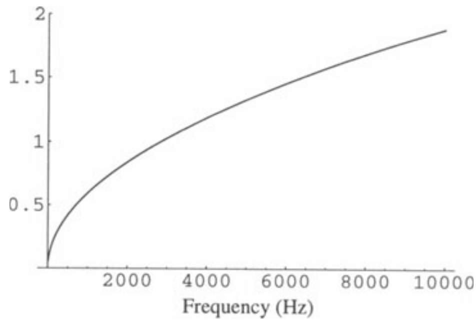

Fig. 6. Number of cycles/cm computed as function of frequency for aluminum. Our proposed criterion for non-quasistatic mode is that one half of the wavelength must be smaller than the plate thickness. The lowest allowed frequency for a 1cm thick aluminum plate is seen to be about 720 Hz.

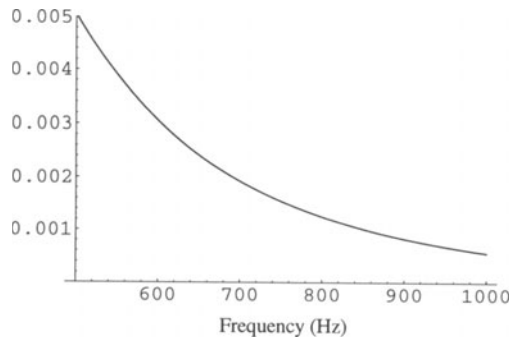

Fig. 7. Fraction of field amplitude after traversing 2 cm in aluminum with respect to the input amplitude. At 720 Hz the fraction is about 0.17 % which is the same at the lowest allowed frequency for any plate thickness.

and $\vec{E}$ leads $\vec{H}$ in phase by $\pi/4$. Substituting the expressions of a and b, which include the characteristics of diffusing field, into the expression of the damped wave and with the phase difference between $\vec{E}$ and $\vec{H}$, the following equations are obtained:

$$\vec{E} = \vec{E}_o e^{-\frac{x}{\delta}} e^{i(\omega t - \frac{x}{\delta})}, \quad \vec{H} = \vec{H}_o e^{-\frac{x}{\delta}} e^{i(\omega t - \frac{x}{\delta} - \frac{\pi}{4})}.$$

The phase velocity of these waves is obtained to be

$$v_p = \frac{\omega}{\beta} = (\frac{2\omega}{\sigma\mu})^{\frac{1}{2}}$$

which, for a magnetic wave of 1 MHz in aluminum, is 531 m/sec.

It may be reasonable to define a certain frequency component as quasi-static mode if its wavelength is larger than the total pathlength in a medium. This means that the plate thickness should be at least equal to one half of the wavelength defining the lowest allowed frequency for wave propagation. For aluminum, using $\sigma = 3.54 \times 10^7$ siemens/m, one can estimate certain useful properties. Fig. 6 shows the number of cycles per/cm in aluminum as a function of frequency. Taking the plate thickness as 1 cm, the frequency that corresponds to 0.5 cycle/cm is found to be 720 Hz which is the lowest allowed frequency for wave propagation. Fig. 7 shows H/H_o after the wave traverses 2 cm as a function of frequency. At f = 720 Hz, the return amplitude is less than .2% of the input amplitude assuming a total reflection at the boundary.

Even though the results of Fig. 6 and 7 are based on the plate thickness of 1 cm, it can be readily shown that the above fraction of the return amplitude is the maximum for plate thickness. For a good conductor, as shown above, $\lambda = 2\pi\delta$ and the total attenuation of the field amplitude after traversing a complete wavelength will be $e^{-2\pi}$ which is about 1.87×10^{-3} for the lowest allowed frequency for a given plate thickness and it decreases with frequency as shown in Fig. 7.

Computing the total amplitude of the return signal requires integration of H/H_o from the lowest frequency allowed to infinity, which can be safely replaced by 2 kHz for aluminum. This computation based on realistic experimental conditions will be included in the future work.

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